# Magnetization profiles for a $d = 2$ Ising strip with opposite surface fields



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# Magnetization profiles for a $d = 2$ Ising strip with opposite surface fields

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**Abstract.** We obtain an exact solution of the $d = 2$ infinite ($n \times \infty$) Ising strip with surface fields of opposite signs $h_1 < 0$, $h_2 \equiv +\infty$. Earlier predictions by Parry *et al* are confirmed of the crucial importance of wetting for a system confined between two parallel walls that exert competitive surface fields. The transition of the magnetization profile $m(z)$ from the partial wetting regime below the critical wetting temperature at a single wall ($n = \infty$) with a surface field $h_1 T_w(h_1)$, to the soft-mode single phase in the temperature range $T_{c,b} > T > T_w(h_1)$, where $T_{c,b}$ is the critical temperature of the bulk system, is described in detail. A scaling ansatz is verified for the singular part of the surface excess free energy, $\sigma^{sing}$ and for the profile, $m(z, T, n, h_1)$. The modification of the magnetization profile near one wall, due to wetting, is studied. Near the $(-)$ wall at $T \approx T_w(h_1)$ the profile is found to be a linear function of the scaled distance but its slope is different than for the case of perfect asymmetry system $h_1 = -h_2$ hence these two cases belong to the different universality regime.

Recently, there has been an increasing interest in systems confined by walls. At the same time there are only a few exactly solvable models available, which can verify phenomenological or approximate theories of a rich variety of phenomena arising in these systems. In this work we present exact calculations for an Ising magnet confined between two parallel plates or walls which exert surface fields $h_1$ and $h_2$ on the respective surface layers. Previous exact theoretical treatments [1] have focused on an Ising magnet and on surface fields of the same sign, $h_1 h_2 > 0$. Here we consider an Ising magnet confined by walls which exert surface fields of the opposite sign, $h_1 h_2 < 0$. The results and conclusions will be applicable to simple fluids, with the usual caveats. For fields of opposite signs $h_1 h_2 < 0$ and $h_1 = -h_2$, a novel phase behaviour has been predicted by Parry and Evans [2, 3] on the basis of the mean-field-type calculations. They have found that if one wall favours liquid (spin up), while the other favours gas (spin down), phase equilibria of the system are strongly influenced by wetting, in contrast to the case of similar walls ($h_1 h_2 > 0$). For a finite distance between the walls $n$, the coexistence (or pseudo-coexistence in $d = 2$) of two confined phases does occur for $T < T_{c,n}$ where the temperature $T_{c,n}$ is determined, for large $n$, by the (critical) wetting properties of the confining walls rather than the bulk critical properties. $T_{c,n}$ is shifted *below* the wetting temperature $T_w(h_1)$ of the semi-infinite system with a surface field on the wall $h_1$. The shift of the $T_{c,n}$ is determined by a length scale pertinent to critical wetting: $T_w(h_1) - T_{c,n} \sim n^{-1/\beta_s}$, where $\beta_s$ is the critical exponent that describes the growth of the wetting film at a single wall. Above $T_{c,n}$ the existence of a *single* soft-mode phase has been predicted [2]. This phase is characterized, for $h = 0$ and large $n$, by the magnetization profile resembling that of the free up-spin–down-spin interface located at the centre of the slit and the large transverse correlation length $\xi_\parallel \sim n^{2/(3-d)}$ for $d < 3$. These predictions confirmed the previous suggestions by Brochard-Wyart and de Gennes [4] and were confirmed in turn by Monte Carlo simulations [5, 6]. Although Parry *et al* have specialized to perfect asymmetry ($h_2 = -h_1$), they have argued that their results are not all restricted to this special case. For other work based on mean-field models, and related work see [7–9].

In this paper we consider a $d = 2$ Ising magnet confined between parallel walls. The system is infinite in the $x$-direction and has a finite width $n$ in the $z$-direction. We study the case where the surface fields are of opposite signs but there is *no* perfect asymmetry in a system. We choose here the surface fields so that wall (2) will be completely wet by the (+) phase for all $T > 0$ ($h_2 = +\infty$) and wall (1) will be completely wet by the (−) phase only above a certain (critical) wetting temperature $T_w(h_1)$. For such a system we give an analytical transfer matrix (TM) solution taking the column–column transfer matrix in a direction parallel to the walls. Owing to the exact diagonalization of TM [10] we calculate the surface excess free energy (per unit area) and the magnetization profiles of that system in a wide range of temperature. We discuss the shape of the profiles below $T_w(h_1)$ and for $T_w(h_1) \geqslant T \geqslant T_{c,b}$ and compare it to the case of perfect asymmetry of surface fields. Our calculation of the magnetization profiles confirms the existence of the *single* soft-mode phase. For large $|h_1|$, $T_w(h_1)$ can lie arbitrarily far below the $T_{c,b}$, so that the soft-mode phase can extend over a wide temperature range. We have earlier studied in detail the magnetization profiles and the Fisher–de Gennes scaling of $m(z, T, L)$ for $h_2 = -h_1 = +\infty$ for which $T_w(h_1) = T_w(h_2) = 0$ [11].

We also answer the following important questions: what happens to the system when there is no asymmetry in the opposing fields; does the predicted [2, 4, 12] behaviour persist beyond mean-field approximation, when strong capillary and critical fluctuations are present? The case of two-dimensional systems is the ultimate test of the validity of the above predictions, since we know that in this low dimension, fluctuations are particularly strong.

In order to study the role of wetting for the location of pseudo-critical temperature $T_{c,n}$, we verify the proposed [2, 3] scaling ansatz in the vicinity of $T_w(h_1)$. It is different from that in the standard finite-size scaling near the critical point. The argument leading to this ansatz is based on the idea that the wetting film thickness $l$ is a dominant scale in the system as $T \to T_{c,n}$ where $T_{c,n}$ lies very close to the wetting temperature $T_w(h_1)$. Hence the scaling near $T_w(h_1)$ is formulated in terms of the exponent $\beta_s$ that describes the growth of the wetting film thickness $l$. We verify the scaling form for the surface excess free energy (per unit area) $\sigma^{\text{sing}}$

$$\sigma^{\text{sing}} = n^{-\tau}\Omega(nt'^{\beta_s}) \qquad h = 0 \tag{1}$$

where $t' = |(T_w(h_1) - T)/T_w(h_1)|$ and $\Omega$ is the scaling function. Exponent $\tau$ is equal to $(2 - \alpha_s)/\beta_s$ but for $d < d_c$ (= 3), $\tau$ reduces to $2(d - 1)/(3 - d)$ when making use of hyperscaling, $2 - \alpha_s = (d - 1)\nu_\parallel$ and of the capillary-wave relationship, $\nu_\perp = (3 - d)\nu_\parallel/2$ together with $\beta_s = \nu_\perp$. We follow the standard definitions of the exponents, e.g. such as used in [3]. The index ‘s’ refers to the ‘surface’. Note that verifying (1) with $\tau = 2$, we are checking more than scaling ansatz alone; indirectly we confirm the relationships for the critical exponents.

As can be seen from our results, the presence of the other wall severely modifies the magnetization profiles near the (−) wall. At the critical wetting temperature $T_w(h_1)$ fluctuations manifest themselves strongly causing the long-ranged surface perturbation [12].

The surface layer magnetization should scale as

$$\Delta m_1^{\text{sing}} \equiv m_{1,n} - m_{1,\infty} \sim n^{-\tau+1/\beta_s} \qquad h = 0 \qquad T = T_w(h_1) \tag{2}$$

which was derived [12] as a direct consequence of (1). We check the above scaling law with $\tau = 2$ and $\beta_s = 1$ in $d = 2$ to see that $\Delta m_1^{\text{sing}}$ decays as $n^{-1}$, which is slower than the known decay $n^{-2}$ in $d = 2$ at the bulk critical temperature. The fluctuations responsible for this behaviour have no bulk counterpart; *interfacial* fluctuations lead to the effects found here. The generalization of this result to distances $z$ near the walls predicts the linear behaviour near the $(-)$ wall at $T_w(h_1)$ [12]. This agrees with our result although the slope of our profile is not a number calculated in [12] for the perfect asymmetry case $h_1 = -h_2$.

Let us now specify the model and describe the solution. It is the $d = 2$ Ising model in the geometry of infinitely long strip $(M \times N, N \to \infty)$ with the fixed $(+/-)$ boundary conditions: $\sigma(x, z = 0) = -1$ and $\sigma(x, z = M) = +1$ for all $x = 1, \ldots, N$ $(N \to \infty)$ and with a nearest-neighbour ferromagnetic interaction $J_1$ and $J_2$, parallel and perpendicular to the $x$ axis, respectively. We set $J_1 = J_2 = J$ and, in addition, one row of modified bonds, $J = a_0 J$, $0 < a_0 < 1$ is introduced parallel to the $x$ axis, between $z = 0$ and $z = 1$ rows, so that the surface field $h_1 = -a_0 J$ acts on the row $z = 1$ and the surface field $h_2 = J$ acts on the row $z = M - 1$ (equivalent to $h_2 = +\infty$ acting on the row $z = M$). The transfer matrix (TM) acts along the $x$ axis, i.e. along the wall(s). We diagonalize the symmetrized TM, $V = V_1^{1/2} V_2 V_1^{1/2}$, with

$$V_1 = \exp\left[-K^* \sum_{1}^{M-1} \sigma_m^z\right] \tag{3}$$

$$V_2 = \exp[(a_0 K)\sigma_0^x \sigma_1^x] \exp\left[K \sum_{2}^{M} \sigma_{m-1}^x \sigma_{2m}^x\right] \tag{4}$$

where $\sigma^i (i = x, y, z)$ are the Pauli matrices, $K = \beta J$ where $\beta = 1/kT$ and $\tanh K^* = \exp(-2K)$ defines the dual coupling $K^*$. The advantage of our formulation lies in the explicit appearance of the weakened coupling $a_0 K$ in $V_2$. The solution of the eigenproblem with spinor analysis [13], similar to [14, 15], with the additional use of projection operators [11], leads to the eigenvectors $|L\rangle = f_{l_1}^\dagger, \ldots, f_{l_j}^\dagger |0\rangle$ with an *odd* number of different anticommuting Fermi operators $f_k^\dagger$, $0 < k \leqslant M$ acting on the 'vacuum' $|0\rangle$ determined by $f_l|0\rangle = 0$ for all $l$. $|L\rangle$ has the eigenvalue

$$\Lambda = \Lambda_0 \exp[-\gamma(\omega_{l_1}) - \cdots - \gamma(\omega_{l_j})] \tag{5}$$

where $\Lambda_0 = \exp\left(\frac{1}{2}\sum_{i=0}^{M} \gamma(\omega_i)\right)$ and $\gamma(\omega) \geqslant 0$ is the Onsager's function [16],

$$\cosh\gamma(\omega) = \cosh 2K^* \cosh 2K - \sinh 2K^* \sinh 2K \cos\omega\,. \tag{6}$$

In addition $\gamma_0 = 0$. Each eigenvalue is still doubly degenerate. The condition for $M$ allowed wave numbers $\omega_j$ is obtained from the solution of the associated eigenvalue problem, details of which will be reported elsewhere, which has the form

$$\mathrm{e}^{\mathrm{i}2(M-1)\omega} = \alpha \mathrm{e}^{\mathrm{i}\delta'(\omega)} \mathrm{e}^{\mathrm{i}\phi(\omega)} \qquad \alpha = \pm 1 \tag{7}$$

where $\mathrm{e}^{\mathrm{i}\phi}$ is defined by

$$\mathrm{e}^{\mathrm{i}\phi(\omega)} \equiv -\mathrm{i}\frac{T + bqz}{Tqz - b} \tag{8}$$

with

$$T \equiv \frac{-\sinh^2 2a_0K}{(\mathrm{e}^{-\gamma} - \cosh 2a_0K)} + \mathrm{e}^{-\gamma}\cosh 2K^* - \cosh 2a_0K \tag{9}$$

$$qz = -\mathrm{i}\frac{\mathrm{e}^{\mathrm{i}\omega}\sinh 2K - \mathrm{e}^{-\gamma}\sinh 2K^*}{\mathrm{e}^{-\gamma}\cos 2K^* - \cosh 2K} \tag{10}$$

and $b = \mathrm{e}^{-\gamma}\sinh 2K^*$. Here $\delta'(\omega)$ is a parameter of Onsager's hyperbolic triangle [14, 16] and has the following factor form:

$$\mathrm{e}^{\mathrm{i}\delta'(\omega)} = (AB)^{-1/2}\left[\frac{(z^2 - A)(z^2 - B)}{(z^2 - A^{-1})(z^2 - B^{-1})}\right]^{1/2} \tag{11}$$

with $z^2 = \mathrm{e}^{\mathrm{i}\omega}$. Here $A^{-1} = \tanh K \tanh K^*$, $B = \tanh K/\tanh K^*$ and the branch of square root is taken such that $\exp \mathrm{i}\delta'(0) = -1$. Let us denote $T_{\mathrm{w}}(h_1)$, the critical wetting temperature, obtained exactly by Abraham [17] for a $d = 2$ semi-infinite square Ising model with a surface field $h_1 = -a_0J$. It satisfies the condition

$$W \equiv (\cosh 2K^* + 1)(\cosh 2K - \cosh 2a_0K) = 1\,. \tag{12}$$

As follows from the behaviour of the function $\delta'(\omega)$ and $\phi(\omega)$, there exists a temperature $T_{\mathrm{w},M}(h_1)$ which lies slightly below $T_{\mathrm{w}}(h_1)$ (for large $M$, $T_{\mathrm{w},M} \sim T_{\mathrm{w}} - \mathrm{constant}/M$); for $T > T_{\mathrm{w},M}(h_1)$ all solutions of (7) or (11) are real but below there exist only $M - 1$ real solutions between 0 and $\pi$ which give non-trivial eigenvectors. All real solutions could be found graphically or numerically from equation (7) rewritten in the form

$$\tan((M-1)\omega) = \tan(\delta'(\omega)/2 + \phi(\omega)/2)\,. \tag{13}$$

For $T < T_{\mathrm{w},M}(h_1)$ the (one) 'missing' root corresponds to $k = 1$, and is found by allowing $\omega_1$ to be imaginary $\omega_1 = \mathrm{i}v$, $v > 0$, $\alpha = 1$. For large but finite $M$, $v$ lies exponentially close to $v'$, where $\mathrm{e}^{-v'} \equiv W^{-1}$,

$$v - v' \sim \mathrm{e}^{-2Mv'}\mathrm{e}^{-2\delta'(\mathrm{i}v')}2W^{-1}\sinh v'\,. \tag{14}$$

Having found $v$ we find $\gamma_1$ from

$$\cosh\gamma_1 = \cosh v_0 + 1 - \cosh v\,. \tag{15}$$

Here $v_0 = 2(K - K^*)$ and is the familiar interfacial free energy per unit area divided by $kT$. For $T_{\mathrm{w},M} < T < T_{\mathrm{c,b}}$ all $M$ solutions for $\omega$ are real and are found from (13). The corresponding $M$ values of $\gamma(\omega)$ are given by (6) and are all greater than $v_0$. For $T < T_{\mathrm{w},M}$ again $\gamma(\omega)$ corresponding to $\omega$ real lie above $v_0$ but $\gamma_1$ which comes from $\omega$ imaginary is less than $v_0$. The full discussion of the behaviour of these roots, depending on all parameters of the strip, will be presented elsewhere. In practical computations equation (13) as well as equation (7) for $\omega_1 = \mathrm{i}v$ have been solved numerically to machine accuracy.

For the average magnetization $\langle\sigma_m\rangle$ in the limit of an infinitely long ($N \to \infty$) strip we obtain using diagonalization of $V$ the following formula:

$$\lim_{N\to\infty}\langle\sigma_m\rangle = -(\mathrm{i})^m\langle 0|f_1\Gamma_0\Gamma_1\cdots\Gamma_{2m}f_1^\dagger|0\rangle\,. \tag{16}$$

This formal expression for $\langle\sigma_m\rangle$ has been transformed into a form suitable for practical computations using the Wick theorem and simplifying the Pfaffian to a determinant of a certain matrix $B$ (in a way similar to that described in [11]).

$$\langle\sigma_m\rangle = -\det(B)\,. \tag{17}$$

This new explicit expression can be calculated and computed.

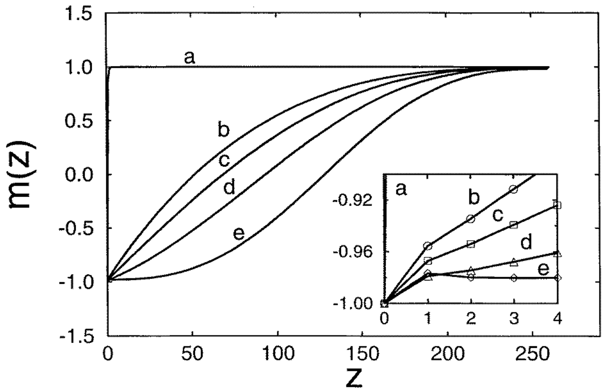


**Figure 1.** A selection of computed magnetization profiles $m(z)$ for fixed $M = 260$, fixed $a_0 = 0.8$ at the $z = 0$ wall and various temperatures in units of $T_c$: (a) $T/T_c = 0.4$; (b) $T/T_c = 0.6201$; (c) $T/T_c = 0.6212$; (d) $T/T_c = 0.623$; (e) $T/T_c = 0.7$. For $a_0 = 0.8$ the wetting temperature $T_w/T_c \approx 0.6212$. The lines are drawn through all 260 points. The inset shows the blown-up portion of $m(z)$ near the $(-)$ wall.

We used $M \in [20, 400]$. Figure 1 shows typical plots of the magnetization profile for fixed width of the strip $n \equiv M - 1$, fixed value of parameter $a_0 = 0.8$ and at different temperatures. Well below $T_w(h_1)$, $\langle\sigma_m\rangle$ is nearly constant equal to $+m^*(T)$, for almost all $m$ except few lattice spaces near the $(-)$ wall,

$$+m^*(T) \equiv +[1 - (\sinh 2K)^{-4}]^{1/8} \tag{18}$$

being the spontaneous magnetization of the $(+)$ phase in the bulk system, shown as curves (a) and (b) of figure 1. Rapid but continuous change of the magnetization profile takes place in the narrow interval about $T_w(h_1)$. Very close to the $(-)$ wall the profile becomes linear; for $T \simeq T_w(h_1)$ this linear behaviour extends quite far from the wall and the slope read off the figure is equal to $\sim 3.6 \pm 0.45$ (curve (c)). The value of this slope is known at present only to accuracy $\pm 0.45$ since it is estimated from the plot. Slightly above $T_w(h_1)$ the derivative $\partial m_1/\partial z$ decreases to 0 and then it changes sign. At the same time there appears an inflexion point and a plateau of the $(-)$ phase near the $(-)$ wall. With the further increase in $T$ the inflexion point gradually moves away from the wall, the film of the $(-)$ phase grows and eventually we get an asymmetric profile (curve (e)) with the interface in the middle of a strip. Also the derivative $\partial m_1/\partial z$ becomes positive (not shown in figure 1). This behaviour may also be obtained by varying the magnitude of $a_0$ at fixed temperature. For our choice of surface fields, below $T_w(h_1)$ for $n \to \infty$ the wall $(+)$ is completely wet and the wall $(-)$ is partially wet. Because $|h_2| > |h_1|$, for finite $n$ below $T_w(h_1)$ almost the whole system is filled with the $(+)$ phase, the wall $(-)$ being partially wet with the $(-)$ phase. Above $T_w(h_1)$ we observe the interface-like profile which, according to the interpretation given by Parry *et al* [2, 3], is characteristic for the *single soft-mode phase* that arises above the pseudo-critical point, $T_{c,n} < T_w(h_1)$, for a system with opposite surface fields [2].

To check the scaling ansatz for the surface excess free energy (per unit area) $\sigma^{\text{sing}}$ given by (1), we have calculated $\sigma(n)$ from

$$\beta\sigma(n) = -\lim_{N\to\infty} (1/N) \log(Z^{+-}/Z^{++}) \tag{19}$$

where $Z^{+-}$ and $Z^{++}$ are partition functions for our system with opposite $(+/-)$ and fixed $(+/+)$ boundary conditions, respectively. In the $N \to \infty$ limit $\sigma(n)$ is equal to $\gamma_1$ given by (15) if $T < T_{w,M}$ or given by (6) if $T_{w,M} < T < T_{c,b}$. Then we have plotted $-n^2\beta\sigma^{\text{sing}}$, where $\beta\sigma^{\text{sing}} \equiv \beta\sigma - v_0$, as a function of $y \equiv nt'$, where $t' = |(T_w(h_1) - T)/T_w(h_1)|$, for fixed $a_0$ and different values of $n$ (figure 2). As can be seen from figure 2, the scaling is very good starting from $y \sim -20$.

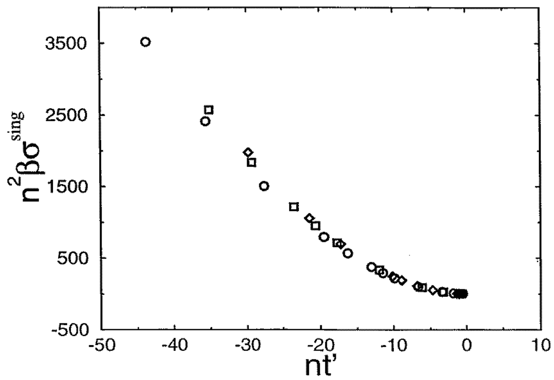


**Figure 2.** Scaling for the surface excess free energy (per unit area), $\sigma^{\text{sing}}$, given by (1). $n^2\beta\sigma^{\text{sing}}$ is plotted as function of $y = nt'$ for fixed $a_0$ and different $n$: ($a$) circles for $n+1 = 100$, ($b$) diamonds for $n+1 = 180$, ($c$) squares for $n+1 = 260$.

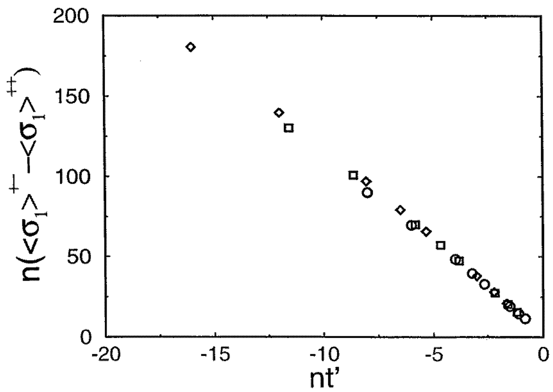


**Figure 3.** Scaling for the surface perturbation given by (2). $n(\langle\sigma_1\rangle^{+/-} - \langle\sigma_1\rangle^{+/+})$ was calculated for fixed $a_0$ and different $n$: ($a$) circles for $n+1 = 100$, ($b$) diamonds for $n+1 = 180$, ($c$) squares for $n+1 = 260$, and plotted against $y = nt'$.

Also, the prediction for perturbation of the surface layer magnetization $\Delta m_1^{\text{sing}} \equiv m_{1,n} - m_{1,\infty}$, (equation (2)) is confirmed for this model as follows from figure 3 where we have plotted $n(\langle\sigma_1\rangle^{+/-} - \langle\sigma_1\rangle^{+/+})$ calculated for fixed $a_0$ and different $n$, against $y = nt'$. $(\langle\sigma_1\rangle^{+/-}$ and $\langle\sigma_1\rangle^{+/+})$ are surface magnetizations for $(+/-)$ and $(+/+)$ boundary condition, respectively.

The scaling relations for the magnetization profile in a parallel plate geometry with opposing surface fields at the fluctuation-dominated critical wetting transition were proposed by Parry [18]. He made an ansatz

$$m(z, T, M)/m^* = \mathcal{M}(zt'^{\beta_s}; nt'^{\beta_s}) \qquad h = 0, T \leqslant T_w(h_1) \tag{20}$$

valid asymptotically for $t' \to 0$ and $n \to \infty$.

To check (20), we plot $(m(z) - m^*)/m^*$, calculated for different $n$, as a function of $z/n$ for fixed $y = nt'$ ($\beta_s = 1$ for a $d = 2$ Ising model) and fixed $a_0$. Scaling is excellent for a wide range of variable $y$ from $y = -10$ up to $y = +10$ thus not only for $T \leqslant T_w(h_1)$ as predicted. Figure 4 shows two scaling functions: ($a$) for $y \sim 0$, i.e. for $T \sim T_w(h_1)$; ($b$) for $y = +10$ where the profile is typical for the single soft-mode phase. For $y \sim 0$ the profile is linear near the $(-)$ wall.

We can compare our results for the scaled magnetization profiles with the available $d = 2$ results from the RSOS and capillary-wave (CW) Hamiltonian [18] model for strip geometry. For $T = T_w(h_1)(y = 0)$ the capillary-wave Hamiltonian for the perfect asymmetry case gives the linear profile across the whole strip. For the case of no perfect asymmetry $|h_1| \neq |h_2|$ the profile is linear only near wall (1) and does not depend on the surface fields

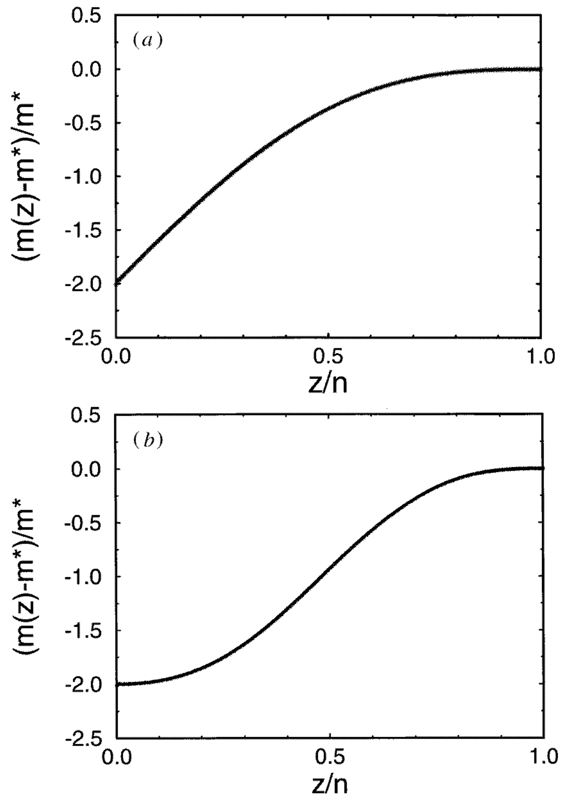


**Figure 4.** Scaling of the magnetization profiles. $(m(z, T, M) - m^*(T))/m^*(T)$ was calculated for different $n$ and fixed $a_0$ and plotted against $z/n$ for the following values of the second variable $y = nt'$: (*a*) $y \sim 0$; (*b*) $y = +10$. (*a*) Shows linear behaviour near 0; (*b*) shows typical single soft-mode profile. All points collapse onto one curve.

$h_1$, $h_2$ [18]:

$$m(z)/m^* = -1 + 2z/n + (2/\pi)\sin(\pi z/n) \qquad T = T_w(h_1)\,. \tag{21}$$

For $T > T_w(h_1)$ in both cases: (i) perfect asymmetry $h_1 = -h_2$ and (ii) no perfect asymmetry $|h_1| \neq |h_2|$ RSOS and CW Hamiltonian models give the profile which is a function of only one scaled variable $z/n$ and does not depend on a particular choice of $h_1$ and $h_2$

$$m(z) = -m^*[1 - 2z/n + \pi^{-1}\sin(2\pi z/n)]\,. \tag{22}$$

Our profiles change their shape with the temperature but sufficiently far from $T_{c,b}$ they perfectly agree with (22). The accuracy of (22) as well as the other properties of the profile in the soft-mode phase were studied earlier [11]. See also [20].

Another scaled profile for $n \to \infty$ was calculated earlier by Abraham [17] above $T_w(h_1)$. That profile describes a single drop on the wall of a semi-infinite system. This implies the limit $M \to \infty$ *first*. The amplitude of the capillary-wave fluctuations is not limited and the scaling variable is $z/\sqrt{s}$ where $s$ is the area covered by drop. In our case $N \to \infty$ first for finite $M$ and the amplitude of capillary waves is cut by the finite width of the strip $n$. For $T \ll T_{c,b}$ but above $T_w(h_1)$ we find expression for the profile like (22) with the scaling variable $z/n$ and not the erf function.

Another predicted [12] consequence of the strong capillary-wave-like fluctuation at the wetting temperature and in the soft-mode phase is the algebraic decay law for the magnetization profiles near one wall. At $T = T_w(h_1)$ the linear behaviour of the profile $m(z)$ near wall (1) was derived [12] from the generalization of the predictions for the surface layer perturbation $\Delta m_1^{\mathrm{sing}}$ to distances $z$ near the walls:

$$\Delta m(z)/m^* \sim c_3(z/n) \qquad T = T_{\mathrm{w}}(h_1) \qquad h = 0 \tag{23}$$

for $z/n \to 0$, $z$ much greater than the bulk correlation length. The same generalization for the temperatures above $T_{\mathrm{w}}(h_1)$ gives [12]

$$\Delta m(z)/m^* \sim c_2(z/n)^3 \qquad T_{\mathrm{c,b}} > T > T_{\mathrm{w}}(h_1) \qquad h = 0 \tag{24}$$

again for $z/n \to 0$, $z$ much greater than the bulk correlation length. The amplitude ratios $c_3$ and $c_2$ have been predicted to be universal numbers which should not change [18] if the spin–spin interactions are modified to model dispersion forces. This followed from analysing the universal scaling functions $m(z)/m^*$ near $T_{\mathrm{w}}(h_1)$ and above, taking into account non-universal metric factors (see [18]). Hence $c_2$ and $c_3$ should be universal for weak- and strong-fluctuation-regime wetting transitions, respectively. Fluids exhibiting van der Waals (dispersion) forces belong to these regimes in $d = 2$.

From the RSOS model with full asymmetry ($h_1 = -h_2$), $|c_3|$ was found [12] to be equal to 2. For $|h_1| \neq |h_2|$, $h_1 h_2 < 0$ the prediction [20] is $|c_3| = 4$ and our slope of $3.6 \pm 0.45$ agrees with this result reasonably well. One must expect the profiles for $h_1 h_2 < 0$ and $h_1 = -h_2$ to be in a different universality class from profiles for $|h_1| \neq |h_2|$ [19].

The shape of a profile for a single inhomogeneous phase above wetting temperature obtained from our calculations agrees perfectly with the profile obtained from RSOS model in both cases (i) perfect asymmetry ($h_1 = -h_2$) and (ii) no perfect asymmetry $|h_1| \neq |h_2|$, thus $c_2$ appears to be universal and in this temperature region both cases belong to the same universality class.

In conclusion, from our exact calculations for an Ising strip with fields of opposite sign it follows that although true criticality does not exist in this $d = 2$ system, the effects of wetting manifest themselves in a manner analogous to that found in higher dimensions. The phase behaviour predicted on the basis of the mean field type calculation supplemented with general considerations [2, 3, 10] survives strong capillary and critical fluctuations. Our choice of surface fields shifts the system away from the coexistence line and allows us to study the behaviour of the single-phase profile, as the wetting temperature is approached. At $T \approx T_{\mathrm{w}}(h_1)$ we have obtained the linear behaviour of the magnetization profile near one wall. This is consistent with the result from RSOS model with ($h_1 \neq -h_2$) but slopes of both lines are different in the two cases ($h_1 = -h_2$ and $|h_1| \neq |h_2|$).

The shape of a profile for a single inhomogeneous phase above wetting temperature obtained from our calculations agrees perfectly with the profile obtained from the RSOS model for both the perfect asymmetry and no perfect asymmetry cases. From that we can conclude that predicted [12] behaviour of the profile near walls is given by (24) with the same universal constant $c_2$ for both cases.

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